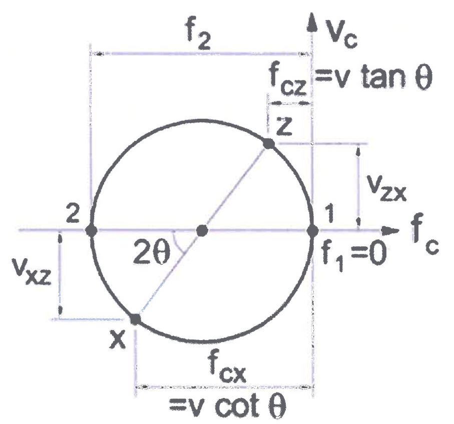
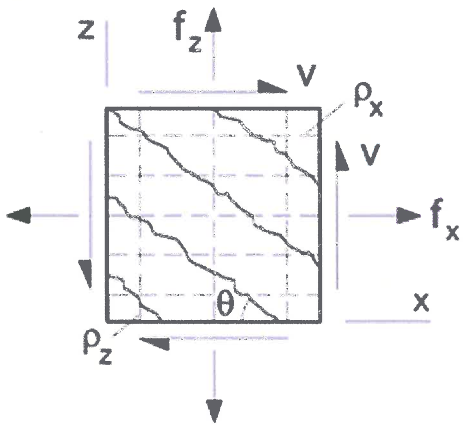
Compression Field Theory



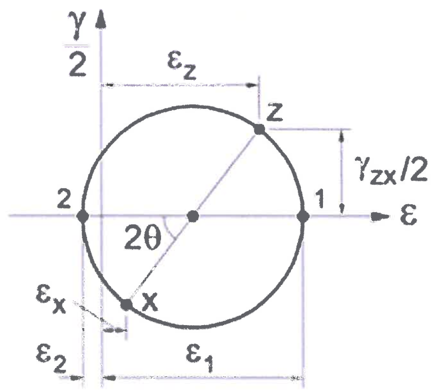


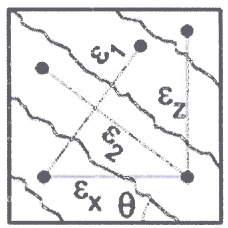
1. $\rho_x f_{sx} = f_x + v \cot\theta$

2. $\rho_z f_{sz} = f_z + v \tan\theta$

3. $f_2 = v\,(\tan\theta + \cot\theta)$

Equilibrium



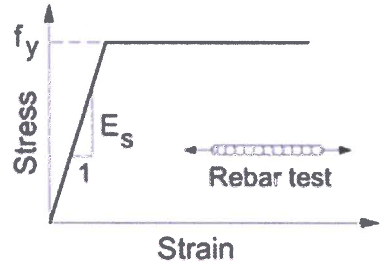


4. $\tan^2\theta = \dfrac{\varepsilon_x + \varepsilon_2}{\varepsilon_z + \varepsilon_2}$

5. $\varepsilon_1 = \varepsilon_x + \varepsilon_z + \varepsilon_2$

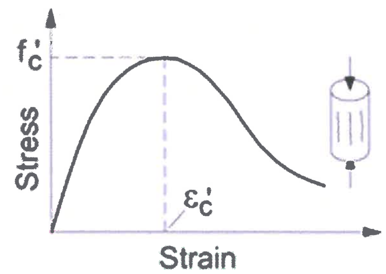
6. $\gamma_{xz} = 2(\varepsilon_x + \varepsilon_2)\cot\theta$

Compatibility



7. $f_{sx} = E_s \varepsilon_x \leq f_{yx}$

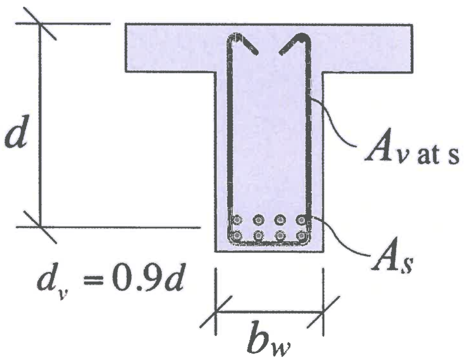
8. $f_{sz} = E_s \varepsilon_z \leq f_{yz}$

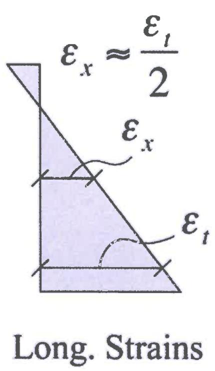


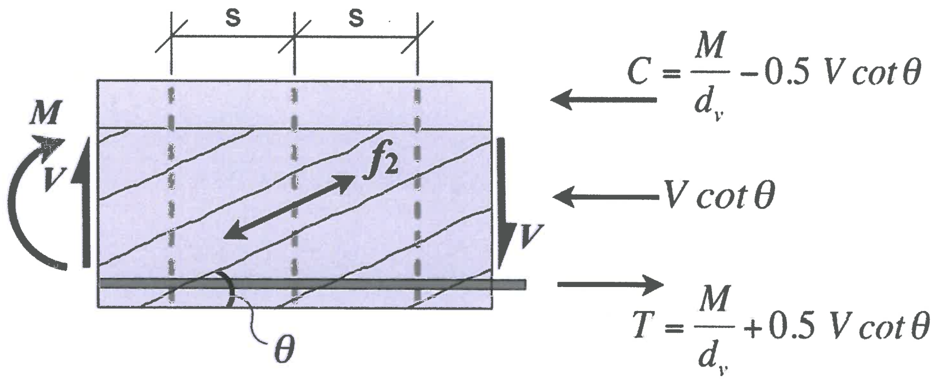
9. $f_2 = f_{2max}\left[2\dfrac{\varepsilon_2}{\varepsilon_c'} - \left(\dfrac{\varepsilon_2}{\varepsilon_c'}\right)^2\right]$

10. $f_{2max} = \dfrac{f_c'}{0.8 + 170\,\varepsilon_1}$

Stress - Strain







Equilibrium

$$v = \frac{V}{b_w d_v} \qquad (1)$$

$$v = \rho_z f_{sz} \cot\theta \qquad (2)$$

$$f_2 = v(\tan\theta + \cot\theta) \qquad (3)$$

$$\rho_{xs} f_s = v\left(\frac{M}{Vd_v} + 0.5\cot\theta\right) \qquad (4)$$

$$\rho_{xs} = \frac{A_s}{b_w d_v} \qquad \rho_z = \frac{A_v}{b_w s}$$

Strains and Stresses

If $f_s < f_y$ then $\varepsilon_x = \frac{v}{2E_s\rho_{xs}}\left(\frac{M}{Vd_v} + 0.5\cot\theta\right)$ (5)

Assuming $\varepsilon_2 = 2\times10^{-3}$ then

$$\varepsilon_1 = \varepsilon_x + \left(\varepsilon_x + 2\times10^{-3}\right)\cot^2\theta \qquad (6)$$

$$f_{2max} = f'_c / \left(0.8 + 170\varepsilon_1\right) \qquad (7)$$

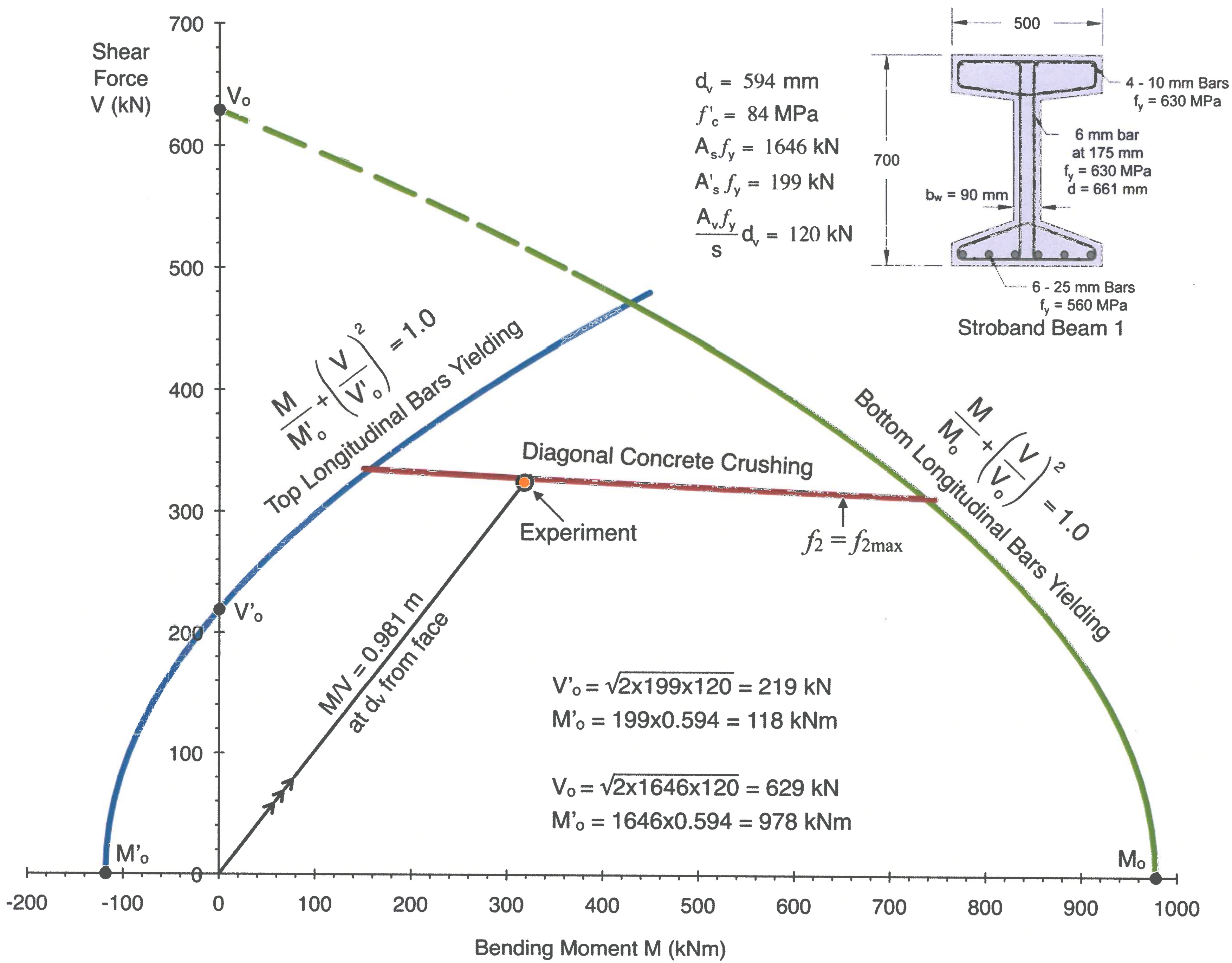
$$\varepsilon_z = \left(\varepsilon_x + 2\times10^{-3}\right)\cot^2\theta - 2\times10^{-3} \qquad (8)$$

$$v = \sqrt{\rho_z f_{yz} f_{2max} - \left(\rho_z f_{yz}\right)^2} \qquad (9)$$

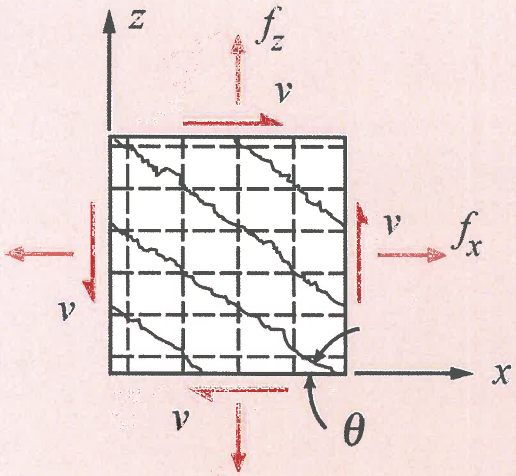
If $f_s = f_y$ $\quad V = V_o\sqrt{1 - M/M_o}$ (10)

$$V_o = \sqrt{2A_s f_y \times \frac{A_v f_{yz}}{s} d_v} \qquad (11)$$

$$M_o = A_s f_y d_v \qquad (12)$$



Shear Force V (kN)
Bending Moment M (kNm)
Vo
V'o
M'o
Mo
d_v = 594 mm
f'c = 84 MPa
As fy = 1646 kN
A's fy = 199 kN
Av fy / s dv = 120 kN
500
700
4 - 10 mm Bars
fy = 630 MPa
6 mm bar
at 175 mm
fy = 630 MPa
d = 661 mm
bw = 90 mm
6 - 25 mm Bars
fy = 560 MPa
Stroband Beam 1
M/M'o + (V/V'o)^2 = 1.0
Top Longitudinal Bars Yielding
M/Mo + (V/Vo)^2 = 1.0
Bottom Longitudinal Bars Yielding
Diagonal Concrete Crushing
Experiment
f2 = f2max
M/V = 0.981 m
at dv from face
V'o = √2x199x120 = 219 kN
M'o = 199x0.594 = 118 kNm
Vo = √2x1646x120 = 629 kN
M'o = 1646x0.594 = 978 kNm



Equilibrium:

Average Stresses:

1. $f_x = \rho_x f_{sx} + f_1 - v \cot\theta$

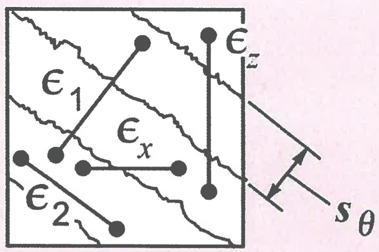
2. $f_z = \rho_z f_{sz} + f_1 - v \tan\theta$

3. $v = (f_1 + f_2)/(\tan\theta + \cot\theta)$

Stresses at Cracks:

4. $f_{sxcr} = (f_x + v \cot\theta + v_{ci} \cot\theta)/\rho_x$

5. $f_{szcr} = (f_z + v \tan\theta - v_{ci} \tan\theta)/\rho_z$



Geometric Conditions:

Average Strains:

6. $\tan^2\theta = \dfrac{\epsilon_x + \epsilon_2}{\epsilon_z + \epsilon_2}$

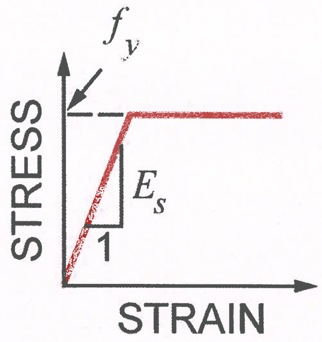
7. $\epsilon_1 = \epsilon_x + \epsilon_z + \epsilon_2$

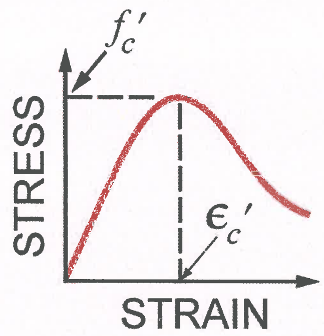
8. $\gamma_{xz} = 2\,(\epsilon_x + \epsilon_2) \cot\theta$

Crack Widths:

9. $w = s_\theta \epsilon_1$

10. $s_\theta = 1 \Big/ \left(\dfrac{\sin\theta}{s_x} + \dfrac{\cos\theta}{s_z} \right)$





Stress-Strain Relationships:

Reinforcement:

11. $f_{sx} = E_s\, \epsilon_x \le f_{yx}$

12. $f_{sz} = E_s\, \epsilon_z \le f_{yz}$

Concrete:

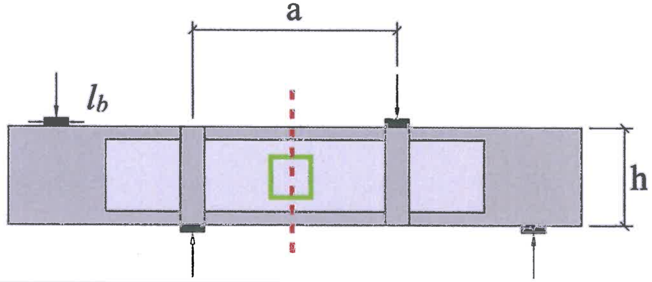
13. $f_2 = \dfrac{f_c'}{0.8 + 170\epsilon_1} \left[2 \dfrac{\epsilon_2}{\epsilon_c'} - \left(\dfrac{\epsilon_2}{\epsilon_c'} \right)^2 \right]$

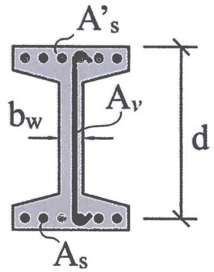
14. $f_1 = 0.33\sqrt{f_c'} \Big/ \left(1 + \sqrt{500\, \epsilon_1}\right)$ MPa

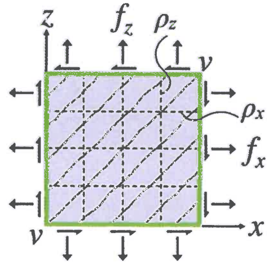
Shear Stress on Crack:

15. $v_{ci} \le \dfrac{0.18\sqrt{f_c'}}{0.31 + \dfrac{24\, w}{a_g + 16}}$ MPa, mm

Analysis Method Using Single Membrane Element







STEP 1)

Determine the shear span-to-depth ratio, a/d

STEP 2)

Determine ρ_x and ρ_z

$$\rho_x = \frac{2A_s}{b_w d_v}$$

$$\rho_z = \frac{A_v}{b_w s}$$

- $d_v = 0.9d$
- If $a/d < 2.75$ then,

$$\rho_z = \frac{A_v}{b_w s}\left(\frac{a/d}{2.75}\right)$$

STEP 3)

Determine the applied stresses,

M/V taken at d_v from face of load or support,
- if clear span < $2d_v$ then take at midspan

$$\frac{f_x}{v} = \frac{M}{Vd_v}$$

$$f_x = \frac{A_p f_{po} + N}{A_g}$$

$$\frac{f_z}{v} = 0$$

- If $\frac{A_v f_y}{b_w s} \leq 0.08\sqrt{f'_c}$ then, $\frac{f_x}{v} = 2\frac{M}{Vd_v}$
- If $A'_s < A_s$ then, $\frac{f_x}{v} = \frac{M}{Vd_v} \geq 1.0$
- If $a/d < 2.75$ then,

$$\frac{f_z}{v} = \frac{d}{a}\left(\frac{2.5}{0.6 + 1.5a/d} - 0.5\right) \leq 0$$

STEP 4)

Determine the crack spacing, s_x and s_z

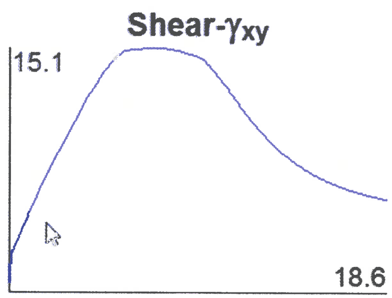
$$s_x = s_{ze} = \frac{35d_v}{15 + a_g}$$

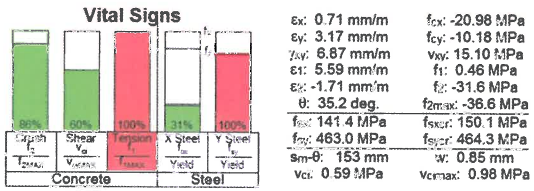
$$s_z = 50{,}000 \text{ mm}$$

- If $\frac{A_v f_y}{b_w s} \geq 0.08\sqrt{f'_c}$ then, $s_{ze} = 300$ mm and, s_z = auto

STEP 5)

Conduct MCFT analysis of membrane element using program Membrane (M2K)





Example (M2F Specimen):

$a = 1278$ mm $\quad d = 564$ mm

$a/d = 2.27$

$d_v = 0.9d = 508$ mm

$b_w = 82$ mm

$f'_c = 75.4$ MPa

$A_s = A'_s = 3050\text{ mm}^2$

$A_v = 100\text{ mm}^2 \quad s = 50$ mm

$f_y = 463$ MPa

$$\rho_x = \frac{2A_s}{b_w d_v} = 14.64\%$$

$$\rho_z = \frac{A_v}{b_w s}\left(\frac{a/d}{2.75}\right) = 2.44 \times 0.83 = 2.01\%$$

$$0.08\sqrt{f'_c} = 0.695\text{MPa}$$

$$\frac{A_v f_y}{b_w s} = 11.3 \text{ MPa} > 0.695 \text{ MPa}$$

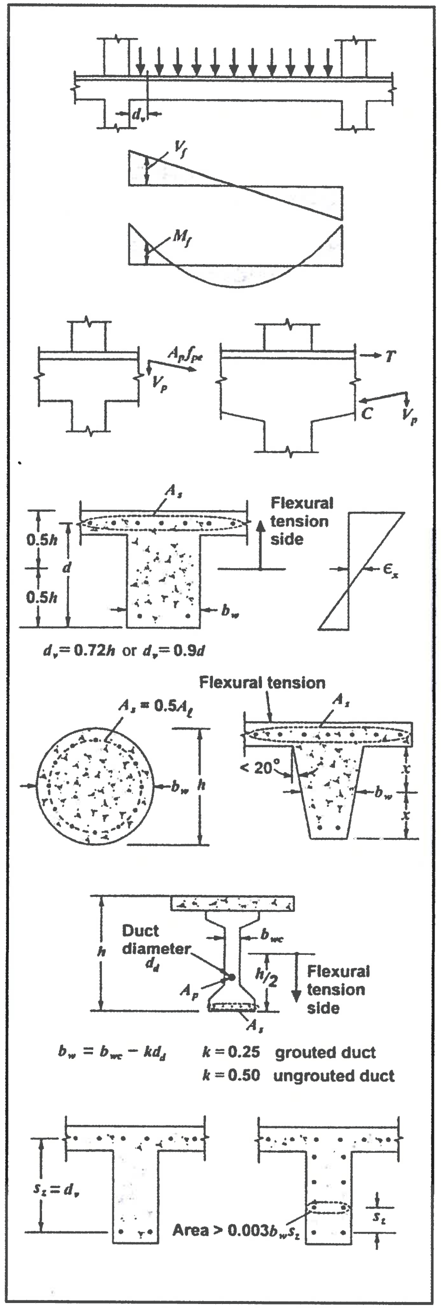
$$\frac{f_x}{v} = \frac{M}{Vd_v} = 0 \quad \text{through contraflexure}$$

$$\frac{f_z}{v} = -0.0547$$

$s_x = 300$ mm $\quad s_z$ = auto

$v_{m2k} = 15.10$ MPa

$$\frac{v_{exp}}{v_{m2k}} = 0.99$$



Canadian Standards Association A23.3-2014 Sectional Shear Design

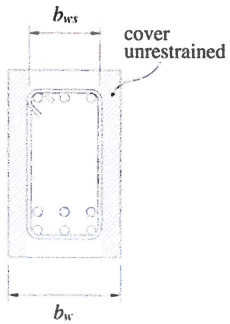
STEP 1

Determine V_f and M_f at section. Take first section d_v from face of support.

STEP 2

For inclined tendons or variable depth members calculate V_p.

STEP 3



Determine b_w and d_v

- If $\dfrac{A_v f_y}{b_w s} > 0.66\sqrt{f'_c}$ and cover unrestrained then take $b_w = b_{ws}$

STEP 4

Determine shear stress

$$v = \frac{V_f - V_p}{b_w d_v}$$

If $v > 0.25\,\phi_c f'_c$ then larger section is needed.

STEP 5

Determine longitudinal strain

$$\varepsilon_x = \frac{M_f/d_v + (V_f - V_p) + 0.5N_f - A_p f_{po}}{2(E_s A_s + E_p A_p)} \geq -0.20\times10^{-3}$$

- Take M_f/d_v as positive not less than $(V_f - V_p)$
- Take N_f as positive if tension, negative if compression
- If numerator negative take $\varepsilon_x = 0$ or add $E_c A_{ct}$ to denominator
- If longitudinal bars are cut off in a flexural tension zone multiply ε_x by 1.5
- For standard prestressing f_{po} may be taken as $0.7 f_{pu}$

STEP 6

Determine θ and β

$$\theta = 29 + 7000\varepsilon_x \qquad \beta = \frac{0.40}{(1+1500\varepsilon_x)}\cdot\frac{1300}{(1000+s_{ze})} > 0.05$$

- For sections containing at least minimum transverse reinforcement,
  $s_{ze} = 300$ mm, If $s > 600$ mm $s_{ze} = (s - 300)$
- Otherwise

$$s_{ze} = s_z\frac{35}{15+a_g} \geq 0.85 s_z$$

Where a_g is maximum aggregate size for coarse aggregate.
If $f'_c \geq 70$ MPa, take $a_g = 0$

STEP 7

Determine required amount of stirrups A_v/s

$$V_c + V_s + V_p \geq V_f$$

$$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$$

where $\sqrt{f'_c} \leq 8$ MPa

$$V_s = \phi_s\left(\frac{A_v}{s}\right) f_y d_v \cot\theta$$

- Check minimum $\dfrac{A_v f_y}{b_w s} \geq 0.06\sqrt{f'_c}$
- Check spacing $s \leq 0.7\, d_v$
- if $v > 0.125\,\phi_c f'_c$, then $s \leq 300$ mm and $s \leq 0.35\, d_v$

$\phi_c = 0.65$, $\phi_s = 0.85$, $\phi_p = 0.90$

STEP 8

Check tensile capacity of longitudinal reinforcement

$$F_{lt} = M_f/d_v + 0.5N_f + (V_f - V_p - 0.5V_s)\cot\theta$$

Factored resistance of longitudinal reinforcement on flexural tension side must not be less than F_{lt}.